Efficient Training of Low-Curvature Neural Networks

IE 663 Course Project

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Members

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Presentation Outline

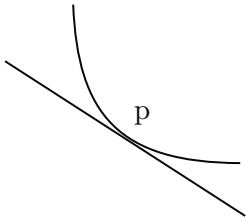
Introduction

Motivation of the Study

- Deep Neural Networks(DNN) are non-linear by nature, but excess non-linearity is undesirable.
- Highly nonlinear models can have a high degree of sensitivity to small changes in the input, which can lead to large changes in the output. This affects the adversarial robustness of the model.
- Highly non-linear models can also suffer from exploding and vanishing gradient problems.
- This paper addresses the excess non-linearity issue using the idea of curvature and introduces training of low curvature Neural Networks.

Curvature

- In mathematics, the curvature is the amount by which a curve deviates from being a straight line.
- Intuitively, it is a measure of how much a curve or surface is bending at a particular point.
- Hessian norm at that point provides a measure of curvature at that point.
- Curvature $C_f(x) = 0 \quad \forall x \in \mathbb{R}^d \iff f$ is linear.



Curvature in ML context

- In ML, curvature measures the non-linearity of a model.
- A common measure of the curvature of machine learning models is via Hessian norms

$$||\nabla_x^2 f(x)||_2$$

Background

Adversarial Robustness

- Adversarial vulnerabilities of neural networks are a widely recognized phenomenon whereby the addition of small, imperceptible amounts of noise to an input can cause deep neural networks to misclassify the input with high confidence.
- Samples generated from adversarial attack methods like the Fast gradient method, Projected gradient descent (PGD) can fool neural networks, although the images look the same to human eyes.
- A go-to method to defend against this vulnerability is adversarial training, which trains models to predict samples correctly generated by adversarial attack methods.
- This approach, however, is computationally expensive and provides no formal guarantees on robustness. The defense against adversarial attacks is an interest of research in the ML community.

Gradient Instability

- Gradient instability occurs due to large hessian norms. [Dombrowski]
- Gradient regularization improves model robustness. [Ros and Doshi-Velez]
- Dombrowski et al. proposed to train low curvature models via softplus activations and weight decay.
- Dombrowski et al. [7] focused on the Frobenius norm of the Hessian

$$||A|| = \sqrt{AA^H}$$

- Author's penalize the normalized curvature.

Lipschitz Layers in Neural Networks

- Ideally with NN we want if input are similar then output should also be similar.
- A function $f : R^M \rightarrow R^N$ is Lipschitz continuous if there is a constant L such that

$$||f(x) - f(y)|| \leq L||x - y||$$

$$\frac{||f(x) - f(y)||}{||x - y||} \leq L$$

  The smallest such L is the Lipschitz constant of f and is denoted $Lip(f)$
- We want constant L to be small as possible.

Lipschitz Layers in Neural Networks

- Property: Let $f = g \circ h$. If g and h are Lipschitz continuous, then f is also Lipschitz continuous with $Lip(f) \leq Lip(g)Lip(h)$.
- Therefore, if we make each component of a neural network such that it is Lipschitz continuous with small Lipschitz constants, the whole neural network will also be Lipschitz continuous with small Lipschitz constants.
- Activation functions and Pooling layers generally have Lipschitz constant $L = 1$
- Spectral Normalization: where linear layers are re-parameterized by dividing by their spectral norm, ensuring that the overall spectral norm of the parameterized layer is 1 which bounds L to be 1.

Training a Low Curvature Neural Network

Measuring Relative Model Non-Linearity via Normalized Curvature

- Common way to define curvature is via Hessain norm, however this measure is sensitive to gradient scaling.
- Scaling a function also scale its hessian.
- To avoid this problem author's proposed normalized curvature.

$$C_f(x) = \frac{||\nabla^2 f(x)||_2}{||\nabla f(x)||_2 + \epsilon}, \quad \epsilon > 0$$

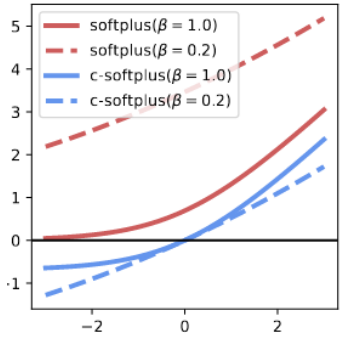
Training a Low Curvature Neural Network

- Directly penalizing the curvature is computationally expensive as it requires calculating the Hessian norm.
- We try controlling the curvature and Lipschitz constant of each layer of a neural network that enables us to control the overall curvature of the model.

Centered Softplus Activation

- Authors propose to use activation functions with minimal curvature.
- softplus function $s(x;\beta) = \frac{log(1+exp(\beta x))}{\beta}$
- curvature of softplus $= \beta(1 - \nabla s(x;\beta))$
- using smaller β ensure lower curvature.

Centered Softplus Activation



(c) Softplus vs Centered-Softplus

- Centered softplus: $s_0(x, \beta) = s(x, \beta) - \frac{log2}{\beta} = \frac{1}{\beta} log(\frac{1+exp(\beta x)}{2})$

Experiments by Author

Experiments by Author

- In experiments authors,
  - evaluates the effectiveness of the proposed method in training models with low curvature
  - evaluate whether low curvature models have robust gradients in practice
  - evaluate the effectiveness of low-curvature models for adversarial robustness
- Dataset Used
  - CIFAR100
  - CIFAR10
- The experiments are performed using NVIDIA GeForce GTX 1080 Tis.
- Implemented in Pytorch
- Baseline model: Resnet18

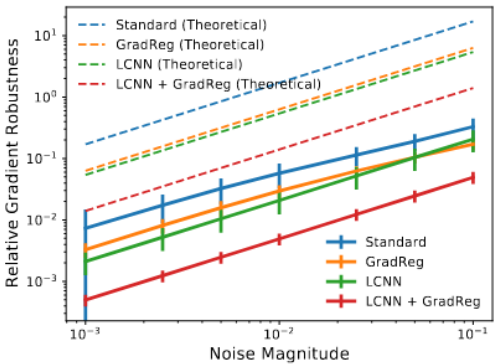
Experiments Results

- Model geometry of various ResNet-18 models trained with various regularizers on the CIFAR100 test dataset.
- Results are averaged across two runs.

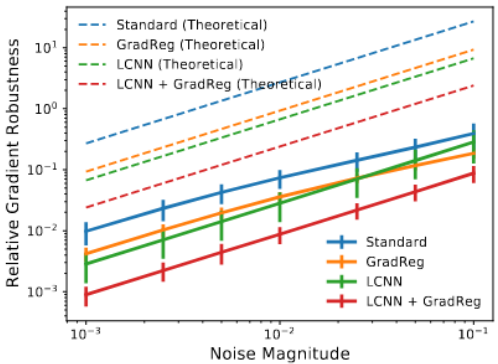
Model	$\mathbb{E}_{\mathbf{x}}\|\nabla f(\mathbf{x})\|_2$	$\mathbb{E}_{\mathbf{x}}\|\nabla^2 f(\mathbf{x})\|_2$	$\mathbb{E}_{\mathbf{x}}\mathcal{C}_f(\mathbf{x})$	Accuracy (%)
Standard	19.66 $\pm$ 0.33	6061.96 $\pm$ 968.05	270.89 $\pm$ 75.04	77.42 $\pm$ 0.11
LCNNs	22.04 $\pm$ 1.41	1143.62 $\pm$ 99.38	69.50 $\pm$ 2.41	77.30 $\pm$ 0.11
GradReg [31]	**8.86** $\pm$ 0.12	776.56 $\pm$ 63.62	89.47 $\pm$ 5.86	77.20 $\pm$ 0.26
LCNNs + GradReg	9.87 $\pm$ 0.27	**154.36** $\pm$ 0.22	**25.30** $\pm$ 0.09	77.29 $\pm$ 0.07
CURE [6]	**8.86** $\pm$ 0.01	979.45 $\pm$ 14.05	116.31 $\pm$ 4.58	76.48 $\pm$ 0.07
Softplus + Wt. Decay [7]	18.08 $\pm$ 0.05	1052.84 $\pm$ 7.27	70.39 $\pm$ 0.88	77.44 $\pm$ 0.28
Adversarial Training [32]	**7.99** $\pm$ 0.03	501.43 $\pm$ 18.64	63.79 $\pm$ 1.65	76.96 $\pm$ 0.26

Experiments Results

Plot showing relative gradient robustness, $\frac{||\nabla f(x+\epsilon) - \nabla f(x)||_2}{||\nabla f(x)||_2}$



(a) Gradient Robustness on CIFAR10



(b) Gradient Robustness on CIFAR100

Experiments Results

Results indicate off-the-shelf model accuracies upon using $l2$ PGD adversarial examples across various noise magnitudes ϵ.

Model	Acc. (%)	$\|\epsilon\|_2 = 0.05$	$\|\epsilon\|_2 = 0.1$	$\|\epsilon\|_2 = 0.15$	$\|\epsilon\|_2 = 0.2$
Standard	77.42 ± .10	59.97 ± .11	37.55 ± .13	23.41 ± .08	16.11 ± .21
LCNN	77.16 ± .07	61.17 ± .53	39.72 ± .17	25.60 ± .32	17.66 ± .18
GradReg	77.20 ± .26	71.90 ± .11	61.06 ± .03	49.19 ± .12	38.09 ± .47
LCNNs + GradReg	77.29 ± .26	**72.68** ± .52	**63.36** ± .39	**52.96** ± .76	**42.70** ± .77
CURE [6]	76.48 ± .07	71.39 ± .12	61.28 ± .32	49.60 ± .09	39.04 ± .16
Softplus + Wt. Decay [7]	77.44 ± .28	60.86 ± .36	38.04 ± .43	23.85 ± .33	16.20 ± .01
Adversarial Training [32]	76.96 ± .26	**72.76** ± .15	**64.70** ± .20	**54.80** ± .25	**44.98** ± .57

Experiments by Team

Experiments by Team

Experimental Setup

- For testing Linux Machine CPU: Intel I3, GPU: Geforce 940MX, 12GB RAM.
- For longer training session Kaggle, GPU: P400, 16GB RAM, was used.

Dataset

- CIFAR10

Programming Setup

- PyTorch
- Default configurations and parameters used by authors were used.

Experiments Results

Model	$\mathrm{E}_x\|\|\nabla f(x)\|\|_2$	$\mathrm{E}_x\|\|\nabla^2 f(x)\|\|_2$	$\mathrm{E}_x C_f(x)$	Accuracy
Standard	5.344159 ± 15.07	668.403381 ± 2161.75	119.779434 ± 86.29	93
LCNNs	2.738105 ± 1.79	11.158964 ± 7.37	4.531949 ± 2.06	64
GradReg	1.444337 ± 1.79	49.747406 ± 126.30	32.834171 ± 19.15	93
LCNN + GradReg	0.900491 ± 0.55	1.988849 ± 1.47	2.161540 ± 1.03	62

Further Investigations

- Analyze robustness towards adversarial attacks

References

- https://towardsdatascience.com/lipschitz-continuity-and-spectral-normalization-b03b36066b0d
- https://arxiv.org/abs/2206.07144

Thank You!