Single Truck Pick Up and Delivery Problem

IE 716 Course Project

Team Name: Dantzig Pickup and Delivery

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Presentation Outline

Introduction

Introduction

- The Traveling Salesman Problem is widely used in a variety of real-life problems in its augmented forms. A class of such problems is the Pickup and Delivery Problem.
- In this class of problems, similar to TSP, we are given a network (in the form of a directed or undirected Graph).
- The nodes are classified as Depot (typically a singleton), Pickup Nodes, and Delivery Nodes. These nodes are associated with quantities of a product.
- Each delivery node requires a given amount of the product, while each pickup node provides a given amount of the product.

Introduction

- The vehicle, starting and ending its route at the depot, needs to collect the product from the pickup nodes and supply it to the delivery nodes.
- Typically, the vehicle has a fixed upper-limit capacity.
- The problem is to find a minimum distance route for the vehicle satisfying all the delivery requirements without ever exceeding its capacity.

Variations of Pickup and Delivery Problem

Variations of Pickup and Delivery Problem

These problems are studied in three main variants:

One commodity Pick Up and Delivery Problem:

- Only one product which needed to be transferred from the pickup nodes to the delivery nodes. No unit has a precise pickup or delivery location.
- Example: when a bank company must move money between branch offices, some of them providing money and the others requiring money; the main office (i.e., the vehicle depot) provides or collects the remaining money.
- Another example occurs when milk must be distributed from farms to factories by a capacitated vehicle, assuming that each factory is only interested in receiving a stipulated demand of milk but not in the providers of this demand.
- This variant was introduced by [Hernández-Pérez and Salazar-González, 2003]

Variations of Pickup and Delivery Problem

Two commodity TSP with Pick up and Delivery

- In these problems the product collected from pickup customers is different from the product supplied to delivery customers.
- Therefore, the total amount of items collected from pickup customers must be delivered only to the depot, and there are other different items going from the depot to the delivery customers.
- An application of PDTSP is the collection of empty bottles from customers for delivery to a warehouse and full bottles being delivered from the warehouse to the customers.
- These problems were introduced by [Mosheiov, 1994]

Variations of Pickup and Delivery Problem

Dial A Ride TSP

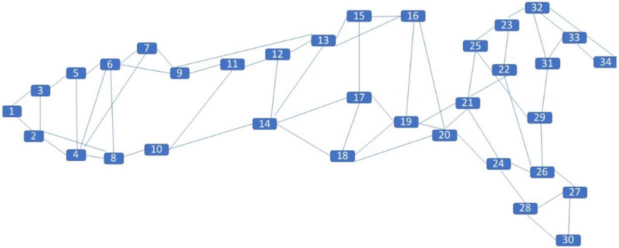
- In these problems, there is a one-to-one correspondence between pickup customers and delivery customers, and each delivery customer must be visited only after the corresponding pickup customer has been visited.
- This is a particular case of TSP with Precedence Constraints.
- An example is post office cargo, where cargo from one PO must be sent to another.
- Food delivery services are another example.
- All these problems can be of single or multiple trucks, single or multiple depots, and combined with the vehicle routing problem.
- In the case of multiple trucks, problems can be further complicated by allowing for transshipment, i.e., cargo can be transferred from one truck to another.
- Dial-a-ride TSP are extremely popular and were introduced by [Stein, 1978].

Toy Example

- Dantzig Pick Up and Delivery Services is a pickup and delivery company situated at Colaba Mumbai.
- One of its regular order is to distribute cargo between 34 stations in Mumbai.
- We assign a truck with a 600 kg payload capacity for the task.
- This is an instance of a single commodity pick-up and delivery problem.
- We solve this problem for three different instances using Solver and compare the results with the standard TSP solution.

Dantzig Pickup and Services

(a) Selected locations from Map



(b) Road Network from Map

Formulation

Let variable $x_{i,j}$ is a binary variable denoting if edge connecting i, j is part of optimal solution or not. Number of these variables is equal to total number of edges.

$$x = \begin{cases} 1 & \text{if edge } (i,j) \text{ is part of optimal solution} \\ 0 & \text{if edge } (i,j) \text{ is not part of optimal solution} \end{cases}$$

Another variable u will track edge number, it starts from 1 and goes till number of nodes. Number of these variables is equal to total number of nodes.
Let $|N|$ denotes total number of nodes and $|E|$ denotes total number of edges.

$$u \in 1, 2, 3, ..., |N|$$

Let take another variable f_a = load of the vehicle going through arc a.

$$f_a \in \mathbb{R}$$

Let $d_{i,j}$ denote distance between cities i and j

Dantzig Pickup and Services I

$$min \sum_{i=1}^{|E|} \sum_{j=1}^{|E|} d_{i,j} x_{i,j}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{i,k} = 1 \quad \forall k \in \{1, 2, 3...., |E|\}, i \neq j \tag{1}$$

$$\sum_{j=1}^{n} x_{k,j} = 1 \quad \forall k \in \{1, 2, 3...., |E|\}, i \neq j \tag{2}$$

$$u_1 = 1 \tag{3}$$

$$u_i - u_j + 1 \leq (n-1)(1 - x_{i,j}) \quad \forall (i,j) \in \{1, 2, 3...., |E|\}^2, i \neq j \tag{4}$$

$$u_i \geq 2 \quad \forall i \in \{2, 3...., |E|\} \tag{5}$$

Dantzig Pickup and Services II

$$u_i \leq |N| \quad \forall i \in \{2, 3...., |E|\} \tag{6}$$

$$\sum_{i=1}^{n} f_{i,k} - \sum_{j=1}^{n} f_{k,j} = q_i \quad \forall k \in \{1, 2, 3...., |E|\}, i \neq j \tag{7}$$

$$0 \leq f_a \leq Q x_a, \quad \forall a \in E \tag{8}$$

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Dantzig Pickup and Services

- $constraint(1)$ denotes from each node there is one incoming edge.
- $constraint(2)$ denotes from each node there is one outgoing edge.
- $constraint(3)$ denotes when $x_{i,j} = 1$ then $u_j = u_i + 1$ ie. j^{th} edge will be labelled 1 more than previous label (i^{th}) label if edge $x_{i,j}$ is selected. This constraint denotes ordering of each node in optimal solution. This constraint is not considered for last edge of path which connect last node with first node to create a cycle.
- $constraint(4)$ denotes first node ordering is always 1
- $constraint(5)$ and $constraint(6)$ are bound constraints on u.
- $constraint(7)$ and $constraint(8)$ are demand constraints and bound constraints on f respectively.

Dantzig Pickup and Services

Assumptions on the model

- The truck has adequate fuel
- The product doesn't deteriorate
- It doesn't matter which time of the day the delivery depot receives the order
- Pickup and delivery at each node is less than the capacity of the truck (taken as 600 kg)

Computational Complexity

Notice that, if the capacity of the truck (Q) is large enough, (bigger than the sum of the delivery demands), it reduces to a TSP problem with maybe some additional constraint. Since TSP is known to be NP-complete, this is an NP hard problem.

Experiments by Team

Experiments by Team

Experimental Setup

- Experiments were done in Google Colab. It provides an Intel(R) Xeon(R) 2.20GHz CPU and 12.7 GB of RAM.

Data

- We collect distance data from Google Maps
- We scraped latitude and longitude information openstreetmap.
- We generated solutions for three different sets of pickup and delivery demand data.

Programming Setup

Pyomo

- Open-source Python-based optimization modeling language.
- Allows users to model and solve complex optimization problems.
- The abstract representation of the optimization model is passed to a solver interface that communicates with the solver.
- The solver interface is responsible for translating the abstract model representation into a format the solver can understand and get solution.

Folium

- It is a Python package that lets us plot maps and visualize.
- The generated maps plots are interactive and customizable.

Experiments by Team

CBC (Coin-or branch and cut) [John Forrest,]

- It is an open-source mixed integer linear programming solver written in C++.
- It relies other parts of the COIN-OR repository. CBC needs a LP solver (CLP) and Cut Generation Library (CGL) for generating cuts.
- CBC uses Branch and Cut Algorithm, along with some heuristics to obtain valid solutions quickly.

GLPK (GNU Linear Programming Kit) [Makhorin,]

- This package is intended for solving large-scale LP and MIP.
- It is written in ANSI C and organized in the form of a callable library.
- It includes branch-and-cut method, primal and dual simplex methods, primal-dual interior-point method.

Experiments by Team

Problem Details

- We solved the same toy problem as described in previous slides.
- Number of Cities: 34
- Number of Delivery Points: 17
- Number of Pickup Points: 16

Model Details

- Total Number of variables: 2346
- Total Number of constraints: 4728

Experiments Results

Experiments Results

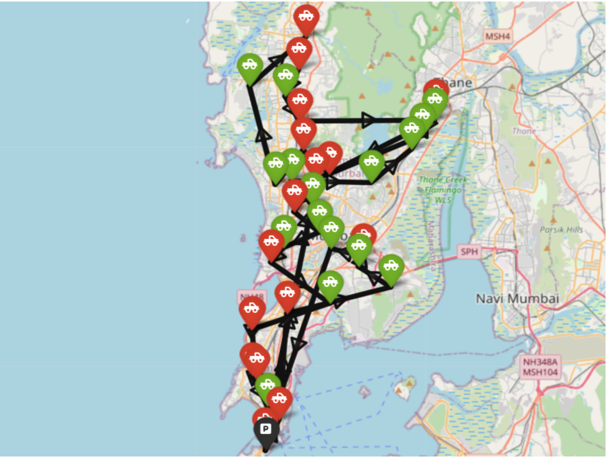
Solution of Problem

- We run problem on test road network as described in previous slide with different demands of each node.
- For some demands we chose for a node solution matches the optimal TSP path.
- Time to solve problem also depends on demands, for some demands it take less than 1 min to get to optimal solution and for some demand values it took 30 min.

Experiments Results

Colaba ⟶ Mumbai GPO ⟶ Dadar ⟶ Anushakti Nagar ⟶ Chembur ⟶ Tilak Nagar Chembur ⟶ Kurla West ⟶ India Post Office Mumbai ⟶ P&T Colony Mumbai ⟶ Andheri Mumbai ⟶ Powai ⟶ Bhandup ⟶ Mulund ⟶ Nahur East ⟶ Seepz Mumbai ⟶ Jogeshwari East Mumbai ⟶ Andheri West ⟶ Kharodi Mumbai ⟶ Borivali Mumbai ⟶ Kandivali Mumbai ⟶ Malad West Mumbai ⟶ Goregaon Mumbai ⟶ Azad Nagar Mumbai ⟶ MIDC Mumbai ⟶ Vile Parle East Mumbai ⟶ Air India Colony Mumbai ⟶ Khar West Mumbai ⟶ Bandra West ⟶ Wadala ⟶ Worli Police Camp ⟶ Worli Naka ⟶ Mumbai Central ⟶ Tulsiwadi ⟶ Kalbadevi ⟶ Colaba

Experiments Results



Experiments Results

Comparision of Solver Time

Solver	Time Taken	Number of sub-problems
CBC	1min 6s	8889
GLPK	1min 1s	8889

Experiments Results

Links to access code and results

- Code can be found at IE716-Team-Dantzig
- Results on 34 cities of Maharastra
  - Result 1
  - Result 2
  - Result 3
  - Result 4

References

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Thank You!